# Multi-frame Image Super-resolution Based on Knife-edges

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*Abstract*— **In this paper, a new method of super-resolution is proposed. Instead of the traditional regularization term of the blur function such as TV regularization and Tichonov, new regularization of the blur function is constructed by using the knife-edge in the low resolution images. The proposed method can reduce the overestimation of the traditional methods. Other two previous super-resolution methods have been taken into the comparison to the proposed method, and results of both synthetic data and real data have shown a much better performance(both in PSNR(peak signal-to-noise ratio ) and visional effects) of the proposed method than the other methods.**

***Keywords- Super-resolution; Knife-edge method; PSF; AM;***

## I. Introduction

High resolution (HR) images provide accurate data in medical diagnostics, segmentation, and remote sensing. As the CCD sampling device has physical shortage on obtaining a HR Image, it is necessary to develop other techniques to enhance the resolution of obtained image. Image Super-resolution is an inverse problem, the purpose of which is to obtain an image with higher resolution from one or several lower resolution images. In this means, image super-resolution can be divided into two types: single-frame image super-resolution and multi-frame image super-resolution[1]. A multiple-frame image provides several images for one scene, which provide more information than a single-frame and are more effective, but the tough problem is the registration of these frames, the result of which largely influences the final results of super-resolution. In this paper, we consider the super-resolution of multi-frame image, and we assume that frames are correctly registered beforehand, where the registration method can be found in [2-4]. The typical Image degradation and down-sampling model is shown as follows [5]:

$$g_k = S(h_k * u) + n_k \qquad (1)$$

where $g_k$ is the $k_{th}$ frame with low resolution, $h_k$ is the corresponding blur function, $n_k$ is the additive noise, and $S$ is the down-sampling operator, which is:

$$S_{X,Y}(f)(x,y) = \int_{x_i}^{x_{i+1}} \int_{y_j}^{y_{j+1}} f(\hat{x},\hat{y})\, d\hat{x}\, d\hat{y} \qquad (2)$$
$$(x,y) \in [x_i, x_{i+1}] \times [y_j, y_{j+1}]$$

Where $X = \{x_i\}$ and $Y = \{y_j\}$ are countable sets dividing the continuous domain of the image into blocks, the size of each block depends on the physical sampling device. After down-sampling, the images turn to a block constant function.

Stark and Oskoui proposed POCS [6] which assumes that the low resolution (LR) images are different projection of the HR image. Other methods, like P-G [7] and IBP(iteration back project), all of which are based on frequency extrapolation and projection, but they did not take the blur function into consideration. Sroubek and Flusser proposed Blind Super-resolution (BSR) [5] based on model (1), which is very effective, but the degradations are estimated blindly with assumption that the blur function of each frame is co-prime[1] . The major problem of BSR is that regularization terms of degradations do not perform stably and the overestimation of the degradations widely exists, which straightforwardly influence the final result. Almeida [8] tried a priori that the prime edge of the image is very sparse, and iteratively changed the weight parameter of this priori to reach a correct estimation of both the image and the blur function, but further work still need to be done on the parameter choice.

Knife-edges widely exist in images, which can provide information of the blur functions. Edge method is a typical way of extracting point spread function (PSF) of the image system using the Knife-edges. A basic idea is to extract the PSFs from the LR images, and perform it as a constraint on the blur function $h_k$ in (1) as the extracted PSFs provide part of the information of the blur function.

In this paper, we proposed a method of super-resolution using the extracted PSFs from the LR images as a constraint of the blur function. To the author's knowledge, it is the first time we perform super-resolution using knife-edges.

## II. Methodology

In order to solve $u$ from (1), constraints must be added to ensure the uniqueness of the solution. The most common way is to formulate an energy function for minimization. The acquisitive energy function is:

$$E(u,h)=\sum_{k=1}^{K}\| S(u*h_k)-g_k \| +\alpha R[u]+\beta\sum_{k=1}^{K}Q[h_k] \qquad (3)$$

where K is the number of LR frames, $h$ is the integral representation of all the blur functions $h_k$, term $\sum_{k=1}^{K}\| S(u*h_k)-g_k \|$ is formulated to reduce noise, and $R[u]$ and $Q[h]$ are the regularization terms of both $u$ and $h$, the construction of which will be discussed in II-A and II-B.

### A. Regularization term R[u]

$R[u]$ is a smoothing term which based on the assumption that most parts of the image are smooth, which also reduces the noise. Tichonov is a classic regularization term which has the form $R[u]=\int_\Omega |\nabla u|^2 d\Omega$, and it smoothes all over the image including edges as it assumes that the image $u$ is continuous everywhere; but this poor assumption does not accord with the discontinuity of the image in the area of sharp edges, rendering a poor result. Total variation (TV) is a function, the gradient of which can also be defined in the areas of discontinuities, which has the form:

$$R[u]=\int_\Omega |\nabla u| d\Omega \qquad (4)$$

According to Sroubek and Flusser [1] who adopt the scheme of half-quadratic regularization scheme of German [9], we can iteratively minimize the term by introducing an auxiliary variable $v$, and regularization term of $u$ can be written in the following way:

$$R_\varepsilon[u,v]=\frac{1}{2}\int_\Omega \phi(v|\nabla u|^2+\frac{1}{v}) \qquad (5)$$

The minimization is reached when $v=1/|\nabla u|$, then $R_\varepsilon[u,v]=\int_\Omega |\nabla u| d\Omega$. For numerical reasons, it is essential to restrict $v$ in a close set $C_\varepsilon=\{v:1/\varepsilon\le v\le\varepsilon\}$, then for any initial value the $u^0$ and $v^0$, the minimization of $R_\varepsilon[u,v]$ takes the following 2 steps [1]:

Step 1: $u^n=\arg\min_u R_\varepsilon[u,v^{n-1}]$

Step 2: $v^0=\arg\min_{v\in C_\varepsilon} R[u^n,v]$
$=\min(\max(\varepsilon,1/|\nabla u^n|),1/\varepsilon)$ (6)

### B. Regularization Term Q[h]

Estimation of the blur function of the HR image is vitally important to the final result. As knife-edges widely exist in most of the images, especially in images with complex object, we can obtain the PSF from the LR frame using edge methods, and then construct a strong constraint on the blur function. Based on the assumption that most of the blur functions are separable [10], which is shown as follows:

$$h(x,y)=h_h(x)h_v(y) \qquad (7)$$

where $h_h(x)$ and $h_v(y)$ are 1-dimensional PSF of horizontal direction and vertical direction respectively, and they are absolutely integrabel [10].

On obtaining $h_h(x)$ and $h_v(y)$, we should calculate the partial derivatives of each direction in the area of the knife edge. The area of knife-edge is shown as follows:

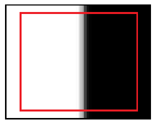

Figure.1 an knife edge area

Each row of the knife-edge is a step function where satisfies:

$$u(x,y)=\begin{cases}1 & x>x_0\\ 0 & x\le x_0\end{cases} \qquad (8)$$

Then following (1) without noise, we obtain its partial derivatives in the horizontal direction of each row:

$$\frac{\partial g(x,y)}{\partial x}=\frac{\partial(S(u*h))}{\partial x} \qquad (9)$$

Consider the separability of $h$ in (7), we obtain:

$$\begin{aligned}\frac{\partial(S(u*h))}{\partial x}&=S(\frac{\partial u}{\partial x}*h)\\ &=S(\delta(x)*h)\\ &=\int_{x_i}^{x_{i+1}} h_h(\hat{x})d\hat{x}\end{aligned} \qquad (10)$$

where $\delta(x)$ is an impulse function. We denote $p_h(x)=<\frac{\partial g}{\partial x}>_y$ and $p_v(y)=<\frac{\partial g}{\partial y}>_x$, where $<\cdot>_y$ denotes an average respect to $y$ and $<\cdot>_x$ denotes an average with respect to variable $x$. We can get $p_h(x)$ and $p_v(y)$ directly by the knife edge in the image, then $Q(h)$ is constructed as follows:

$$Q[h]=\| S_{X,Y}(h_h(x)\cdot h_v(y))-p_h(x)\cdot p_v(y)\| \qquad (11)$$

To be more specific, we always assume that both the horizontal and vertical direction share the same 1-dimensional blur function, which means $p_h(x)=p_v(y)$. Then we need only one knife-edge in practical cases.

We use alternative minimization to minimize both $u$ and $v$, which is shown as follows:

Step 1 $u^n=\arg\min_u E(u,h^{n-1})$

Step 2 $h^n=\arg\min_h E(u^{n-1},h)$ (12)

$\alpha$ and $\beta$ are weight parameters changing with the level of the noise, $\alpha$ should increase when noise increase, whereas $\beta$ should decrease.

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## III. QUALITY MEASUREMENTS

The quality measurement of PSNR and MSE is adopted in this paper to evaluate the quality of the result of super-resolution. The PSNR measurement is shown as follows:

$$\mathrm{PSNR}(u) = \frac{1}{\| u - \tilde{u} \|^2} \quad (13)$$

$$\mathrm{PSNR}(h) = \frac{1}{\| h - \tilde{h} \|^2} \quad (14)$$

And the MSE measurement is shown as follows:

$$\mathrm{MSE}\ (u) = \frac{1}{M \times N} \| u - \tilde{u} \| \quad (15)$$

$$\mathrm{MSE}\ (h) = \frac{1}{M \times N} \| h - \tilde{h} \| \quad (16)$$

where $M$ , $N$ is the height and width of the image respectively, and $u$ is the estimated HR image, $\tilde{u}$ is the original HR image; $h$ is the estimated blurs, and $\tilde{h}$ is the original blurs. But when we come to the real-data experiment, we can only evaluate the result by the vision effects.

## IV. EXPERIMENTAL RESULTS

We tested the proposed method on both synthetically blurred images and real images. We also performed comparisons with two other methods which has been published in [6] and [5].

### A. *Synthetic Experiments*

In this section, we blurred an astronomical image from Prism sensor with 3 different blurs, and then down-sampling them with factors 2. The set of the LR image are shown in Figure.2:

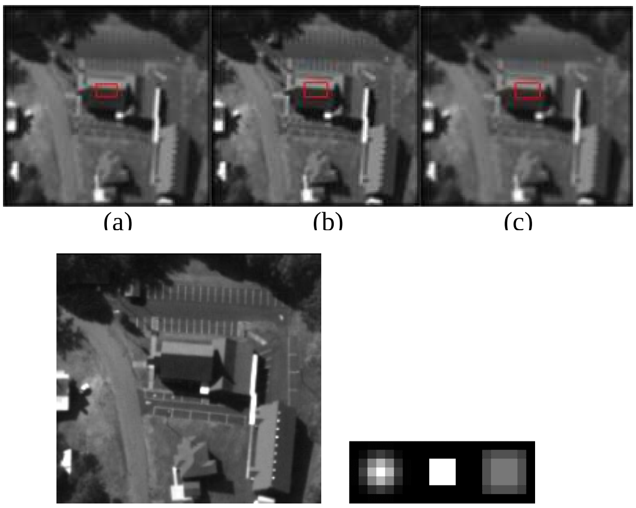


Figure.2 (a) (b) (c) are blurred LR images. with size $99\times93$ ; (d) is the Original HR image with size $192\times180$ and (e) is the 3 synthetic blurs with size $7\times7$, which blurs the original image and render (a), (b) and (c).

Results include bi-cubic interpolation of Figure.2(a), IBP super-resolution, blind super-resolution (BSR) proposed by Sroubek and Flusser and our method, the area containing knife-edge is shown in the red rectangular box in Figure.2 (a), (b) and (c)

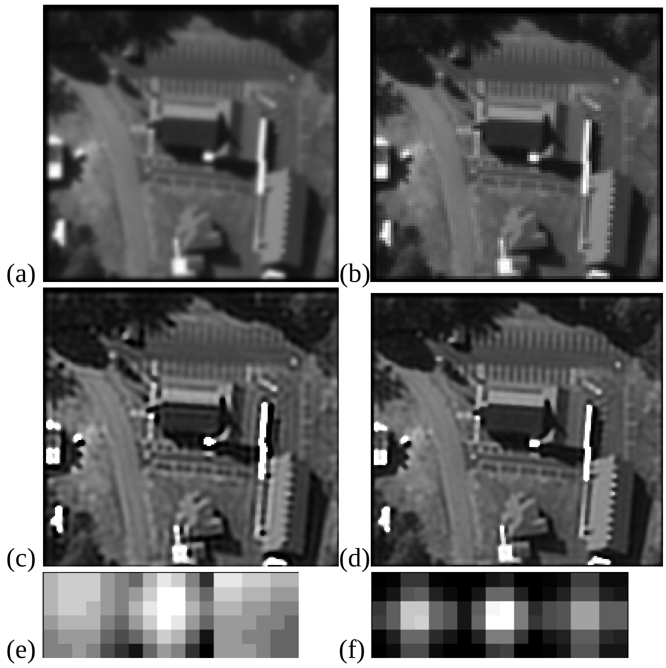


Figure.3 (a) bi-cubic interpolation of Figure,2(a); (b) Result of IBP reconstruction; (c) Result of BSR; (d) Proposed method; (e) Estimated blurs of BSR; (f) Estimated blurs of proposed method.

Results of proposed method shows a better vision on details, and the estimated blur functions of the proposed method is closer to the original one, while those of the BSR is greatly different from the original one. As the original HR image and blur functions exist, the PSNR and MSE of both the image and blurs are shown in TALBLE I:

TABLE I PSNR AND MSE OF IMAGES AND BLURS

| ***Quality Measurement*** | **PSNR(dB)** | **MSE** |
|---|---|---|
| ***Bi-cubic Interpolation*** | 26.8622 | 0.0021 |
| ***IBP*** | 26.8097 | 0.0021 |
| ***BSR*** | 24.9943 | 0.0032 |
| ***Proposed Method*** | 29.4288 | 0.0011 |
| ***Estimated blurs of BSR*** | 30.3592 | $9.2061\times10^{-4}$ |
| ***estimated blurs of Proposed method*** | 32.7495 | $5.3095\times10^{-4}$ |

### B. *Real-data Experiment*

Two Prism images with different angles of view are provided in this section, and the knife-edge we choose for calculate is chosen directly from the two images (the red area within the red rectangular). Before reconstruction, the two of them are registered manually.

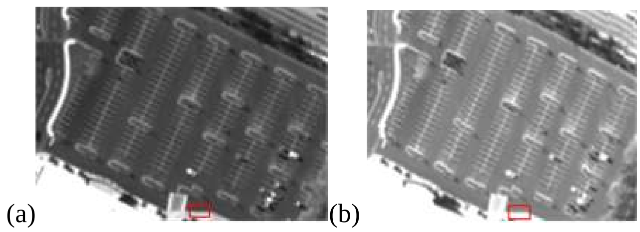
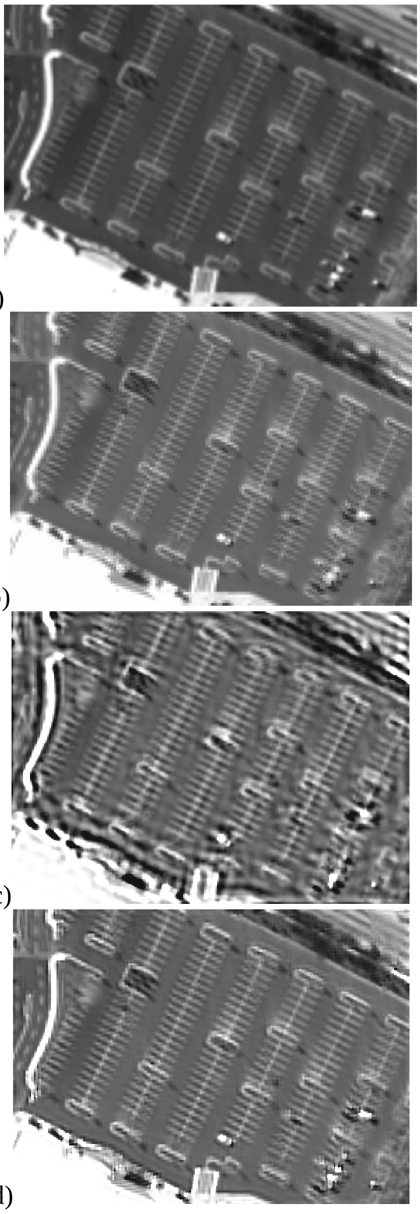


Figure 4 (a) The backward view of the ground; (b) The nadir view of the ground.

Results include bi-cubic interpolation, IBP reconstruction, BSR and the proposed method, which are shown below in Figure 5:



(a) (b) (c) (d)

Figure.5 (a) Bi-cubic interpolation of Figure.4 (a);

(b) IPB; (c) BSR; (d) proposed method.

Visual effects show that the proposed method rendered better results, whereas BSR overestimated the blur function, which engender poor results in Figure.5 (d). As it does not take blur function into consideration for IBP, the result of it is vaguer than the result of the proposed method.

## V. Final Comments

In this paper, we proposed a method of super-resolution utilizing the knife-edge, which contributes a lot in estimating the blur function, then to render a much better results than the traditional methods do. This method is a little refrained as it assumes that the knife-edges can be found in the LR images, which is crucial for reducing the overestimation of blur function. However, the sharp edge which is used in the proposed method is not very strict, and each small area with a relatively high contrast in gray level is suitable. Especially in the remote sensing image, the sharp edge can be easily found in the edge of the ground object. For images with large size, only one edge with several pixels is enough for performing our method. Therefore, it is worthy to find an edge and gain better and more stable results. The knife-edge chosen and registration was done manually, and our further work will focus on the registration of the frames and knife edge automatic detection.

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