A Robust Method of Calculating Point Spread Function from Knife-edge without Angular Constraint in Remote Sensing Images

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Abstract: The traditional knife edge method has the limitation that the chosen edge must be approximately perpendicular or parallel to the sampling direction (ideal edge), which leads to difficulties of choosing usable knife-edge. Focusing on the issue, a new model of calculating PSF based on knife-edges with any slant angles is proposed in this paper. The proposed method is not only equivalent to the traditional one with the ideal edge, but also better than the traditional method with a slant edge in the PSF (Peak Signal to Noise Ratio), MSE (Mean Square Error) and SNR (Signal to Noise Ratio) of the restored image by the extracted PSF. The proposed method is robust to extract PSFs from knife-edges of any slant angles for image restoration, which extends its application. The validity of the proposed method has been reasoned both theoretically and practically, and tested with simulated data and ADS40 data.

Key words: PSF, Levenberg-Marquardt, Knife-Edge method, Wiener filter

1 INTRODUCTION

Image restoration based on point spread function (PSF) is a valid method to improve the quality of image and reduce the influence of optical blurring effect of the sensor (Yu & Zhang, 2002; Khan, et al., 2008). Generally, the PSF can be obtained in the laboratory, but when we are measuring the on-orbit PSF, we should take into consideration the influence of atmosphere turbulence and the motion blur (Gu, et al., 2005; Zhao, et al., 2009). Therefore, Extracting the PSF from the remote sensing images becomes a tough issue in the realm of remote sensing image restoration (Liu, et al., 2004). Currently, the predominant knife-edge method has been widely used: Zeng, et al., (2008) restored the images sets of Satellite CBERS-1 based on the PSF extracted by traditional knife-edge method. Gu, et al.(2005) restored the image sets of CBERS-02B based on the Modulation transfer function (MTF). Liu, et al.(2004) estimates the PSF by using the knife edge of the images of CBERS-1, and deconvolutes the image by using wiener filter, providing crucial value of ameliorating the quality of the image (Zhou, et al., 2009). However, the traditional knife-edge method requires that the knife edge in the image must be perpendicular or parallel to the sampling direction (Choi, 2002) (we call it ideal knife edge). Under most circumstances, few knife edges are the ideal ones. Most of the knife edges have some slope to the direction of the ideal ones (we call it slant knife edges).

An obvious idea is to rotate the knife edge to the ideal direction, but this will destroy the sampling structure of the image, thus we cannot get the accurate PSF, either (Choi, 2002). Facing the limitation of extracting PSF from slant knife edge in the traditional knife-edge method, Zhao, et al. (2009) discussed the influence of the slant angle (the angle between the knife edge and the direction perpendicular to the sampling direction), radiometric calibration, and the contrast of the knife edge on the traditional MTF measurement. The conclusion of the discussion says, when the slant angle is greater than 10°, the estimated PSF curve by the traditional knife-edge method starts to rapidly deviate from the real PSF curve (Zhao, et al., 2009), which renders an extreme depression on the low-frequency area of the image. Chi (2007) had experiments on the simulating MTF of on-orbit satellite, and analyzed the influence of the slant angle of the knife edge on the extraction of the MTF. He proposed to calculate the average of the n edge spread functions to reduce the impact of the slant edge. This works when the slant angle is smaller than arctan(1/n), but as there exists noises, n should at least be 3, but it fails to work when the slant edge is greater than...
18.43°. Till now, there are not too many works on extracting PSF from knife edge of arbitrary slant angle. In practice, there are few ideal knife edges, so it is very significant to have research on how to extract PSF by using knife edges with any slant angles.

With respect to the knife edges with large slant angles, this paper analyzes the cause of inaccuracy when extracting PSF from slant knife edges by using the traditional knife-edge method, and based on this PSF, analyzes the self-convolution relationship of the real PSF. This paper proposes a model of calculating PSF on slant knife edge by constructing non-linear equations, and solves the model by using Levenberg-Marquardt optimal method to calculate the real PSF of the Image system.

2 TRADITIONAL KNIFE EDGE METHOD AND WIENER FILTER

2.1 Traditional knife-edge method

We use function $U(x)$ to denote the step function perpendicular to the knife edge, and $h(x)$ to denote the PSF, then the output image $g(x)$ in the perpendicular direction of the knife edge is

$$\text{g}(x)=U(x)\cdot h(x) \quad (1)$$

where ‘*’ denotes the convolution operator. The partial derivative of $g(x)$ is:

$$\frac{dg}{dx} = \frac{dU}{dx} \cdot h(x) = \delta(x) \cdot h(x) = h(x) \quad (2)$$

where $\delta(x)$ is the unit impulse function. Generally, the PSF of image has the characteristic of separability (Zhao, et al., 2009):

$$h_{x}(x,y) = h_{x}(x) \cdot h_{y}(y) \quad (3)$$

where $h_{x}(x)$ is the PSF with horizontal direction, and $h_{y}(y)$ is the one with the vertical direction. In practical case, as it is hard to find ideal knife edges from image, we always assume the 2 dimensional PSF is symmetrical, which means it has the same 1-dimensional PSF of both the horizontal and vertical direction:

$$h_{x}(x) = h_{y}(x) \quad (4)$$

The process of traditional knife-edge method is shown as follows (Choi, 2002; Wang, 2006):

![Image of traditional knife-edge method process]

The area which contains knife edge is shown as follows:

Choosing the area contains knife edge

Scanning each line and record each edge spread function, find the keen point by interpolation

Fitting each keen point into a straight line via least square method

Calculation average of the each edge spread function

Differentiate

2.2 The model of image blurring and classic wiener filter

After obtaining the PSF of both horizontal direction and vertical direction from Fig. 1, we can calculate the 2 dimensional PSF $h_{x}(x,y)$ according to (3). Let $g(x,y)$ denote the observed image, and $f(x,y)$ denote the high quality image, and $n(x,y)$ denote the additive random noise. Then the observed image could be described as follows (zou, 2001):

$$g(x,y)=f(x,y) \ast h_{x}(x,y) \ast n(x,y) \quad (5)$$

The wiener filter is constructed due to the orthogonality of the error signal and the original signal, its Fourier transform is:

$$G_{u}(\mu,\nu) = \frac{H^{*}(\mu,\nu)}{|H(\mu,\nu)|^{2} + \gamma} \quad (6)$$

where $H(\mu,\nu)$ is the Fourier transform of $h(x,y)$, and $H^{*}(\mu,\nu)$ is the complex conjugate of $H(\mu,\nu)$, $\gamma$ is the ratio of the noise to the signal, which we can assume as a constant (Zou, 2001). Then the Fourier transform of the restored image $F_{u}(\mu,\nu)$ is as follows:

$$F_{u}(\mu,\nu)=G_{u}(\mu,\nu) \cdot G(\mu,\nu) \quad (7)$$

where $G(\mu,\nu)$ is the Fourier transform of the observed image.

3 EXTRACTION OF PSF FROM THE SLANT KNIFE EDGE

3.1 The deficiency of the traditional knife-edge method on the slant knife edge

In the traditional knife-edge method, we need to calculate the partial derivatives of the edge spread function to obtain the 1 dimensional PSF. According to the image blurring model (5) without the influence of the noise, we get:

$$g(x,y)=f(x,y) \ast h_{x}(x,y) = \int_{-\infty}^{\infty} h_{x}(u,v) f(x-u, y-v) dudv \quad (8)$$

Then 1 dimensional PSF is:

$$\frac{\partial g}{\partial x} = \int_{-\infty}^{\infty} h_{x}(u,v) \frac{\partial f}{\partial x}(x-u, y-v) dudv \quad (9)$$

As the PSF is absolutely integrable (Choi, 2002), without loss of generality, we assume:

$$\int_{-\infty}^{\infty} h_{x}(x) dx = 1 \quad (10)$$

The area which contains knife edge is shown as follows:

![Image of knife edge area]

Assuming the value of higher gray level is 1, and that of the lower gray level is 0, and the equation of the line of the knife edge is:

$$x=ay+b \quad (11)$$

Then the original image $f(x,y)$ is:

$$f(x,y)= \begin{cases} 1 & x \geq ay+b \\ 0 & x < ay+b \end{cases} \quad (12)$$

The partial derivative of the sampling direction is:

$$\frac{\partial f(x,y)}{\partial x} = \begin{cases} \delta(x) & x = ay+b \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

Then according to Eq. (3), (9), (10), (12), (13), we get:

$$\int_{-\infty}^{\infty} h_{x}(x,y) \frac{\partial f}{\partial x}(x-u, y-v) dudv = \int_{-\infty}^{\infty} h_{x}(u) \int_{-\infty}^{\infty} h_{y}(v) \frac{\partial f}{\partial x}(x-u, y-v) dudv = \int_{-\infty}^{\infty} h_{x}(x-(ay+b))h_{y}(v) dv \quad (14)$$

The process of traditional knife-edge method is shown as follows.
When $\alpha=0$, the knife edge is perpendicular to the sampling direction, then:

$$ \frac{\partial g(x, y)}{\partial x} = h_i(x-b) \int_0^\infty h_v(v) dv = h_i(x-b) $$ (15)

We can obtain the accurate 1 dimensional PSF of the sampling direction through this ideal knife edge. In practical case, as the sampling system is discrete, when the slant angle is very small such that $\alpha \approx b$ is almost independent of $v$ within the support of $h_i$. In this situation, we can obtain the accurate PSF too.

When $\alpha \neq 0$, though the shape of the extracted PSF curve is correct, this is the self convolution of the real PSF curve, not the accurate PSF. That’s the reason why we cannot obtain the accurate PSF of the image by using the traditional knife-edge method on the slant knife edge. The time complexity of deconvolution is high, but in the practical case, the size of $h_i$ is small, so it is feasible to perform deconvolution.

3.2 Model of calculating PSF based on slant knife edge

The 1 dimensional PSF is a column vector in the discrete form, and the 2 dimensional PSF is a square matrix. According to the separability (3) and symmetry (4) of the 2 dimensional PSF:

$$ Psf = h_i h_i^T $$ (16)

Then the degradation of the image will be expressed as a discrete convolution:

$$ g(m, n) = \sum_{j=0}^{N_j} \sum_{i=0}^{N_i} f(m-j, n-i) \cdot Psf(j, i) $$ (17)

For example, when the length of $h_i$ is 3, $h_i=[u_i, u_3, u_5]$, then:

$$ Psf = \begin{bmatrix} u_i^2 & u_i u_3 & u_i u_5 \\ u_i u_3 & u_3^2 & u_3 u_5 \\ u_i u_5 & u_3 u_5 & u_5^2 \end{bmatrix} $$ (18)

The degraded knife edge is shown as Fig. 3:

3.3 Synthetic experiment

In the synthetic experiment, we convolution a theoretical knife edge with a 5x5 Gaussian window, the theoretical knife edge and the one filtered are shown in Fig. 5:

We adopt the traditional method and the proposed method to extract the PSF curve in Fig. 5 (b):

In Fig. 6, the red thin line with asterisk is the PSF extracted by using the proposed method, and the light thick line with
triangle is the real PSF. The two of them coincide, whereas the PSF curve extracted by using the traditional knife-edge method, which is the blue thin line with circle, deviates from the real PSF. This illustrates that in the ideal case, even if the slant angle is very large, the real PSF still can be extracted by using the proposed method.

![Fig. 5 The Original knife-edge and blurred knife-edge](image)

\[x = \text{X coordinate of the 1 dimensional PSF}
\]

\[x = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
\]

\[y = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
\]

4 ADS40 EXPERIMENT

The data in the experiment is from the ADS40 sensor, the image of which is shown in Fig. 7. We set target of both knife edge with large slant angle and ideal knife edge in the image. As the traditional approach on the ideal knife edge is very mature and widely used, we view the PSF extracted by using the traditional knife-edge method on the ideal knife edge as the accurate PSF, which can be a standard of testing the accuracy of the PSF extracted by using the proposed method on the slant knife edge.

Fig. 7(a) is the panchromatic image of ADS40 sensor, where the circle plain with black-white stripe is the man-made target for experimental purpose. The knife edge target in the Fig. 7(b) is perpendicular to the horizontal sampling direction, from which the PSF extracted by using traditional method is almost accurate. The knife edge in Fig. 7(c) is slant knife edge, the slant angle of which is almost 45°. We adopt the traditional knife-edge method and the proposed method for obtaining PSFs respectively, the results of which are shown in Fig. 8:

Fig. 8 shows the 1 dimensional PSFs, namely the \(h_x\) in (17). We can see in Fig. 8 that the PSF (the red thin line with asterisk) extracted by using the proposed method on slant knife edge is closer to the PSF (light blue thick line with triangle, and we call it the standard PSF) extracted by using the traditional knife-edge method on the ideal knife edge, whereas the PSF (dark blue thin line with circle, and we call it the standard PSF) extracted by using the traditional knife-edge method on the slant knife edge deviate far from the PSF extracted by using the traditional knife-edge method on the ideal knife edge. As we can see in Fig. 8, the PSF extracted by using the proposed method on the slant knife edge and the standard one do not coincide perfectly. The reason for this is that there exists noise during the time when the sensor obtains the reflection light from the ground, and the process of calculating PSF by using L-M method is a process of deconvolution, which might exaggerate the noise to a certain extent. Therefore, the extracted PSF is not as accurate as the one in the synthetic experiment. Even it is not precisely accurate, it is much closer to the standard PSF. The 2 dimensional PSFs are shown in Fig. 9:

We construct the wiener filter using the PSFs shown in Fig. 9, and restored the ADS40, the result of which are shown in Fig. 10.

![Fig. 7 knife-edge Target on ADS40 image](image)

![Fig. 8 Comparison of PSF curve](image)
As the errors of the PSF extracted by using the traditional knife-edge method on slant edge are unacceptable, it renders a depression in the low-frequency area of the image. As we have mentioned above that the PSF extracted by using the traditional knife-edge method on the ideal knife edge is mature and widely used, and it can be viewed as an accurate one. Therefore, for comparing the quality of restoration using different wiener filter constructed from different PSFs, we assume the restored image (Fig. 10 (d)) by using the standard PSF is the definite one, and we evaluate the quality of image through peak signal-noise ratio (PSNR), mean square error (MSE), and signal-noise ratio (Wu, et al., 2008), which is shown in Table 1.

Table 1 Evaluation indicator of the restored ADS40 image in Fig. 10

<table>
<thead>
<tr>
<th>Evaluator</th>
<th>Fig. 10(b)</th>
<th>Fig. 10(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE to Fig. 10 (d)</td>
<td>0.0007</td>
<td>0.0102</td>
</tr>
<tr>
<td>PSNR (dB)</td>
<td>47.3421</td>
<td>40.7374</td>
</tr>
<tr>
<td>MSE (dB)</td>
<td>0.0043</td>
<td>0.0092</td>
</tr>
<tr>
<td>SNR (dB)</td>
<td>17.7474</td>
<td>11.7846</td>
</tr>
</tbody>
</table>

In Table 1, the error between Fig. 10 (b) and Fig. 10 (d) is very slight, and it results from the error of the PSFs. From the visual effect, the Fig. 10 (b) is almost as clear as the Fig. 10 (d), which means, in the realm of image restoration, the PSF extracted by us-
The proposed method on the slant knife edge can replace the standard one. What’s more, the SNR, PSNR and MSE of Fig. 10 (b) are much higher than those of Fig. 10 (e). This illustrates that comparing to the traditional knife-edge method, the PSF extracted by the proposed method perform much better in the image restoration.

5 CONCLUSION

The traditional knife-edge method have limitations of the slant angle of the knife edge, which means that the knife edge must be perpendicular or parallel to the sampling direction. Based on the PSF extracted by using the traditional knife-edge method on slant knife edge, we set up the self convolution relationship of the real PSF. In addition, through setting up non-linear equations, we proposed the calculating model based on the slant knife edge, which can extract acceptable PSF even when the slant angle of the knife edge is very large, and it can be used to perform image restoration very well. Both the synthetic experiment and the ADS40 experiment demonstrate that the PSF extracted by the proposed method on knife edges of arbitrary slant angle can perform as well as the standard PSF in the image restoration, and the proposed method overcomes the limitations of the traditional method. Moreover, it makes the extraction of PSF more stable since it can deal with knife edges with any slant edge, and makes the image restoration based on wiener filter more applicable and feasible. Our proposed method has been used in the large-scale opening remote sensing platform OpenRS and makes reliable products.

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